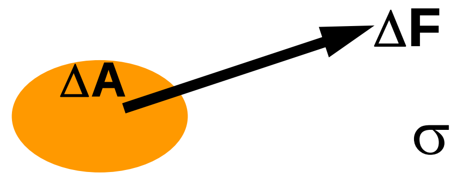
Rock Mechanics

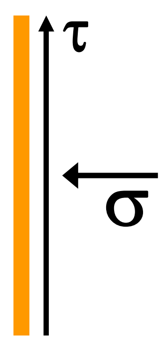
Mechanics and Properties of Rock Material

Stress, Strain and Deformation



$$\sigma = \lim_{\Delta A \to 0} (\Delta F / \Delta A)$$

1. Stress is a property at a point. It is a tensor.



Normal stress σ

$$\sigma = \lim_{\Delta A \to 0} (\Delta N / \Delta A)$$

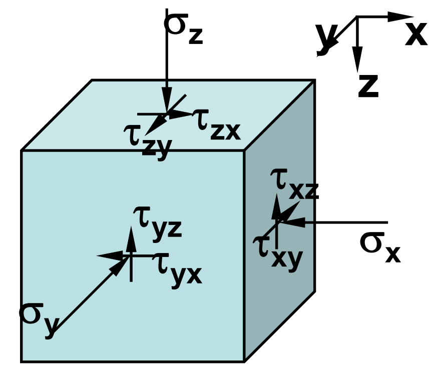
Shear stress τ

$$\tau = \lim_{\Delta A \to 0} (\Delta S / \Delta A)$$

2. There are normal stresses and there are shear stresses.

Stress, Strain and Deformation

3. There are nine stress components on a small cube.



Three normal stresses σ_{xx} σ_{yy} σ_{zz}

Six shear stresses τ_{xy} τ_{yx} τ_{xz} τ_{zx} τ_{yz} τ_{zy}

4. These stress components can be listed out in matrix form.

$$\begin{vmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

Stress, Strain and Deformation

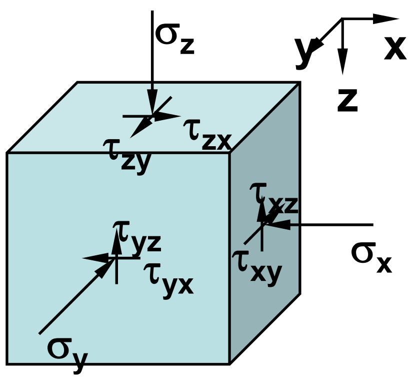
5. Corresponding shear stresses are equal. Hence matrix can be reduced to symmetrical.

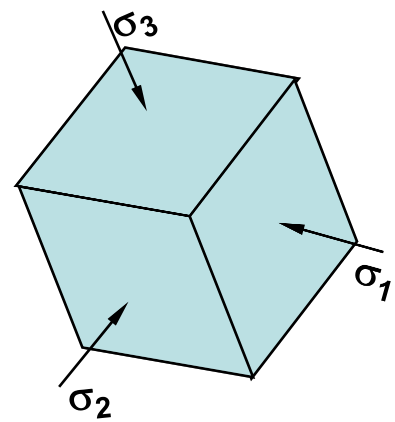
$$\tau_{xy} = \tau_{yx},\ \tau_{xz} = \tau_{zx},\ \tau_{yz} = \tau_{zy}$$

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

Stress, Strain and Deformation

6. There is an inclination of the axes at which all shear stresses disappear (stress transformation). The remaining normal stresses are principal stress





$$\begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}$$

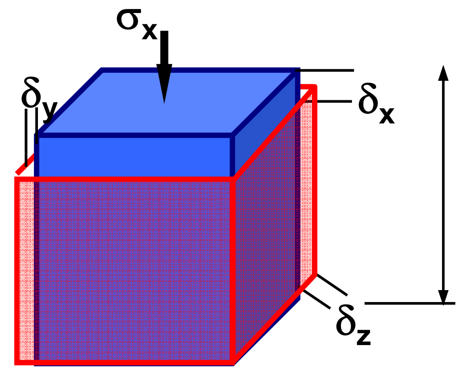
σ_1 = Maximum (major) principal stress.

σ_2 = Intermediate principal stress.

σ_3 = Minimum (minor) principal stress.

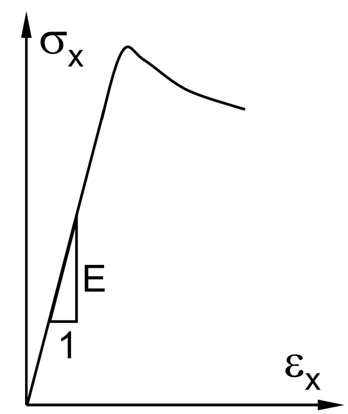
Stress, Strain and Deformation

7. Strains are deformations per lengths caused by stresses. In elastic region, they can be related by the Young's Modulus.



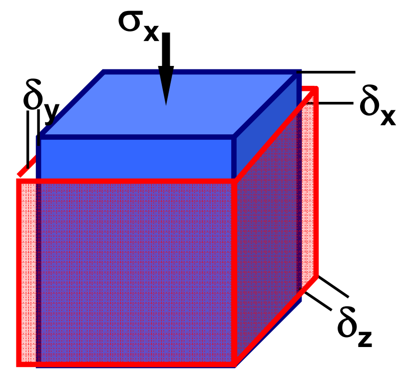
$$\varepsilon = \delta_x / l$$

$$E = d\sigma_x / d\varepsilon_x$$



Stress, Strain and Deformation

8. Strain in stress direction always causes strains in other directions. The ratio of strains is the Poisson's ratio.



$\nu = \varepsilon_y / \varepsilon_x$, $\quad \nu = \varepsilon_z / \varepsilon_x$

Stress, Strain and Deformation

9. Stresses and strains are related by constitutive laws.

$$\begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{vmatrix} \begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{vmatrix}$$

$[\varepsilon] = [S]\,[\sigma]$

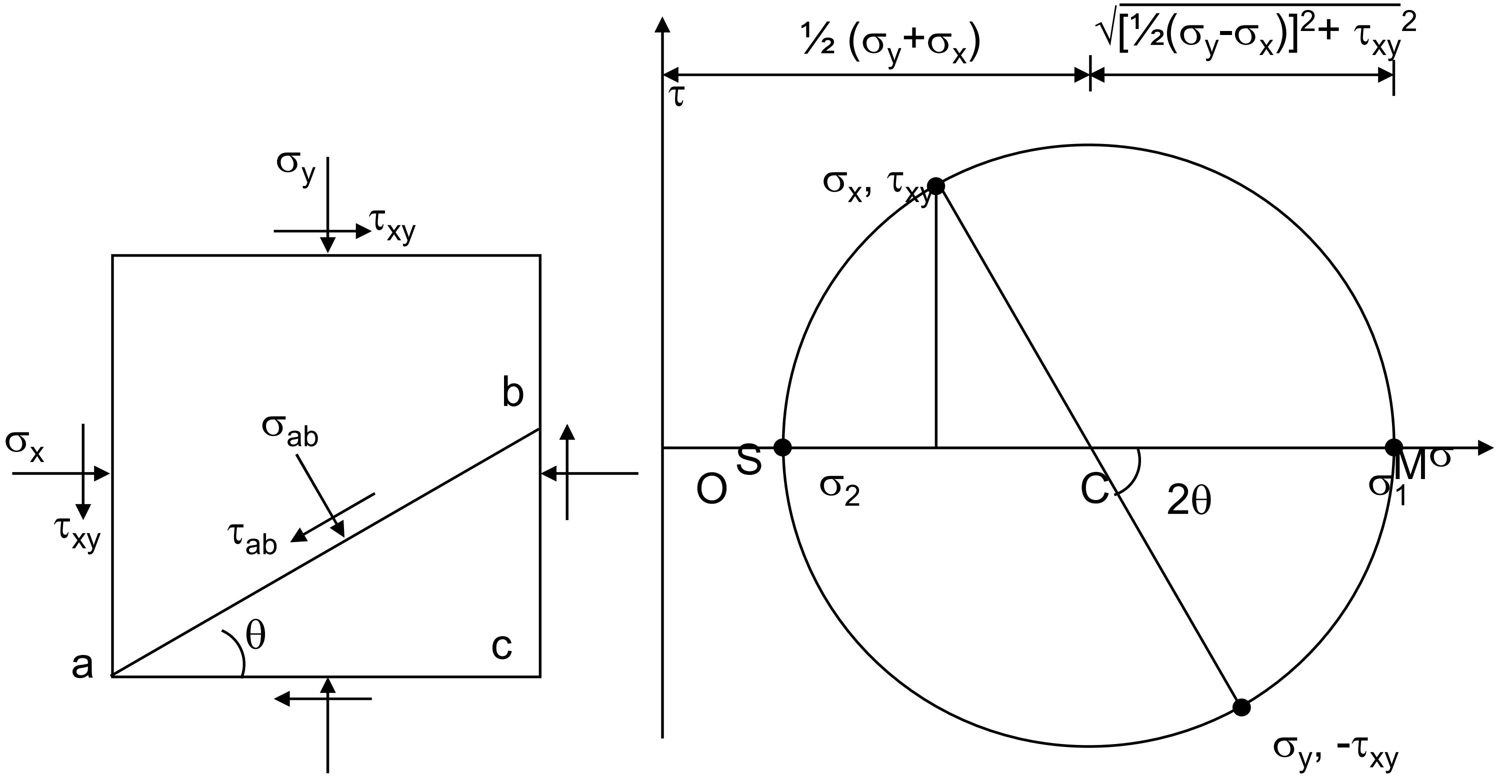
$$\begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} = 1/E \begin{vmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{vmatrix} \begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{vmatrix}$$

$\varepsilon_x = [\sigma_x - \nu\,(\sigma_y + \sigma_z)]\,/\,E$

$\gamma_{xy} = \tau_{xy}\,/\,G$ where $G = E\,/\,[2\,(1+\nu)]$

E = Young's modulus
ν = Poisson's ratio
G = shear modulus

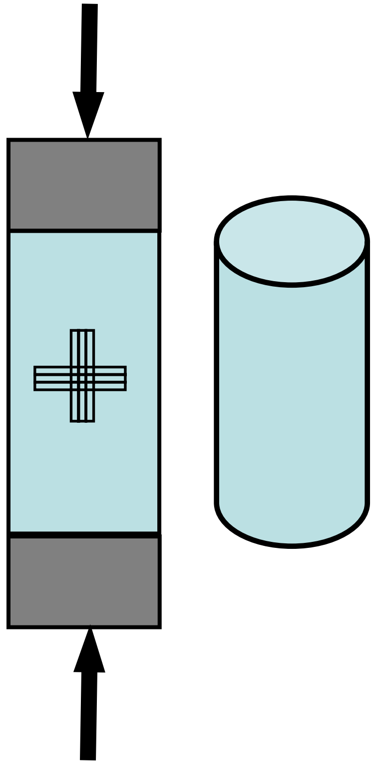
10. Plane stresses and strains can be represented by Mohr circles.



Strength and Deformation

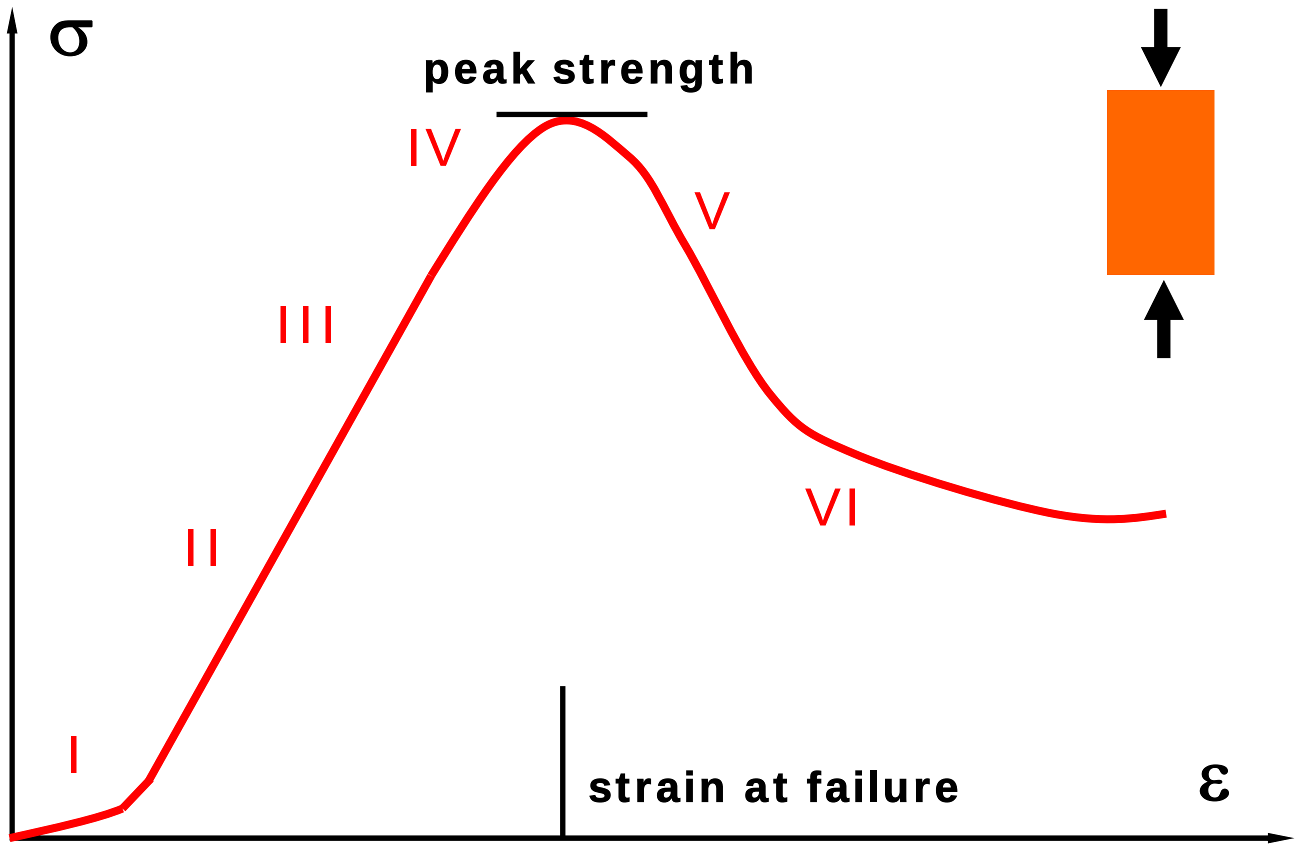
Uniaxial Compression

Uniaxial compressive strength is the ultimate stress a cylindrical rock specimen under axial load. It is the most important mechanical properties of rock material, used in design, analysis and modelling.



Along with measurements of load, axial and lateral deformations of the specimen are also measured.

Strength and Deformation



Stage I – The rock is initially stressed, in addition to deformation, existing microcracks is closing, causing an initial non-linearity of the curve.

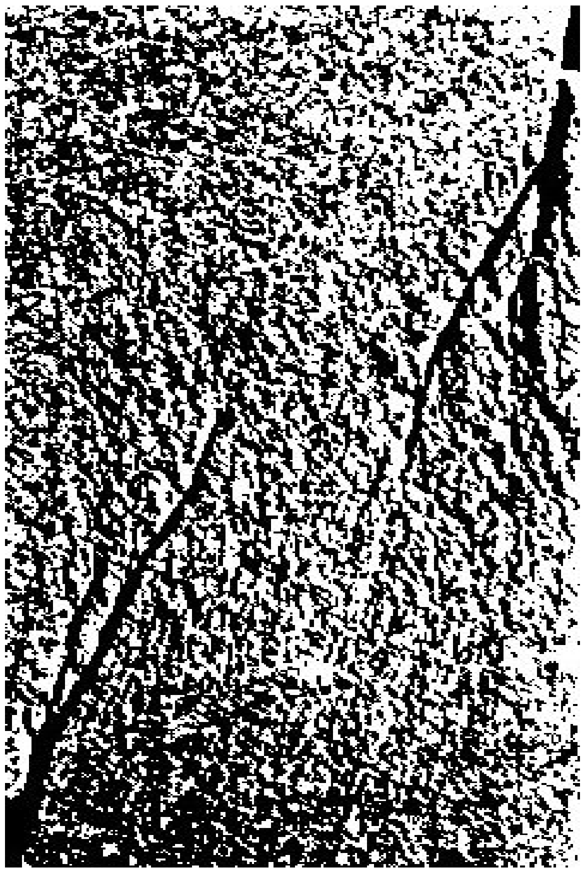
Stage II – The rock basically has a linearly elastic behaviour with linear stress-strain curves, both axially and laterally.

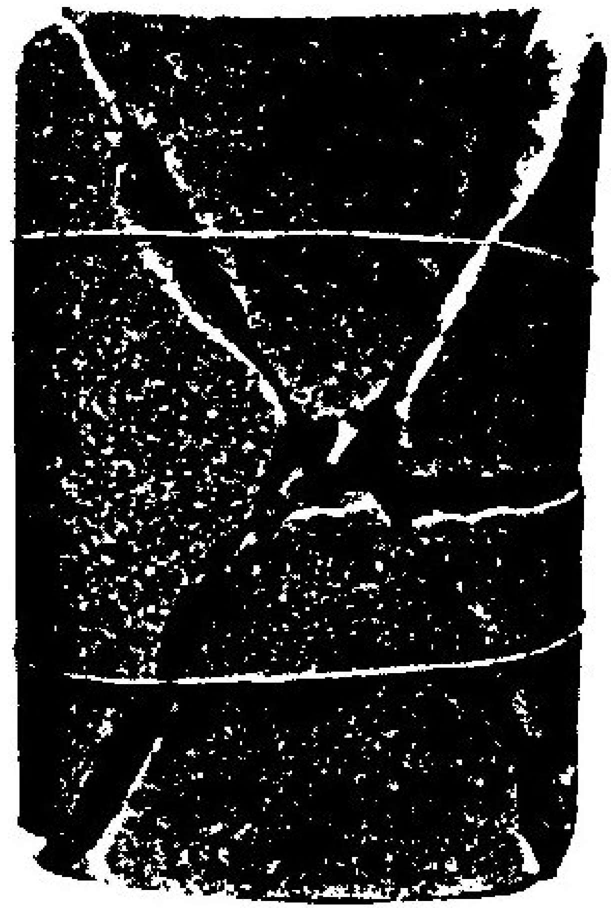
Stage III – The rock behaves near-linear elastic. The axial stress-strain curve is near-linear and is nearly recoverable.

Stage IV – The rock has undergone a rapid acceleration of microcracking events and volume increase.

Stage V – The rock has passed the peak stress, but is still intact, even though the internal structure is highly disrupt. The specimen has undergone strain softening (failure) deformation.

Stage VI – The rock has essentially parted to form a series of blocks rather than an intact structure.


Around peak stress (IV-V)


Post peak stress (VI)

Strength and Deformation

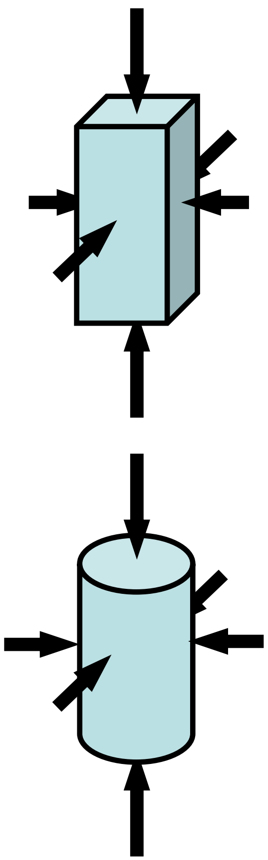
Uniaxial Stress-Strain at and after Peak

Rocks generally fail at a small strain, typically around 0.2 to 0.4%. Brittle rocks, typically crystalline rocks, have low strain at failure, while soft rock, such as shale and mudstone, tend to have relatively high strain at failure.

Most rocks, including all crystalline igneous, metamorphic and sedimentary rocks, behave brittle under uniaxial compression. A few soft rocks, mainly of sedimentary origin, behave ductile.

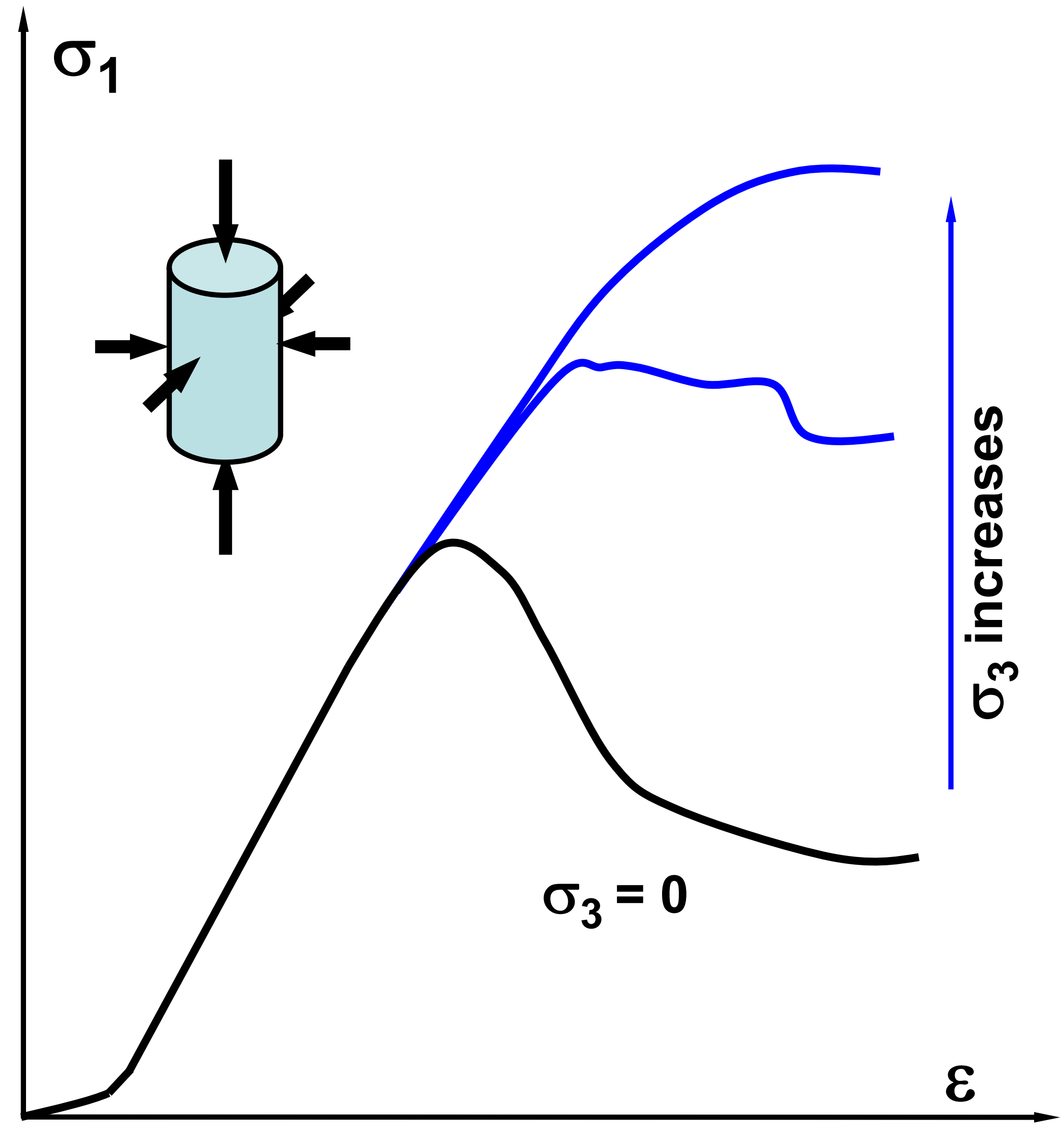
Strength and Deformation

Triaxial Compression



At depth, rock is subjected to axial and lateral stresses (triaxial), and compressive strength is higher in triaxial condition.

True triaxial compression means 3 different principal stresses. It is often simplified by making 2 lateral stresses equal to minor principal stress (axisymmetric triaxial test).



σ1
σ3 increases
σ3 = 0
ε

Strength and Deformation

The behaviour of rock in triaxial compression changes with increasing confining pressure:

(a) Peak strength increases;

(b) Post peak behaviour from brittle gradually changes to ductile.

Stress-strain behaviour at elastic region appears the same as uniaxial compression.

Strength and Deformation

Effect of Intermediate Principal Stress

Axisymmetric triaxial test gives strength without considering the effect of intermediate principal stress (σ_2), and generally under-estimate the strength.

Rock triaxial compressive strength generally increases with σ_2 with a fixed σ_3. When σ_2 is excessively greater than σ_3, the strength may start to decrease.

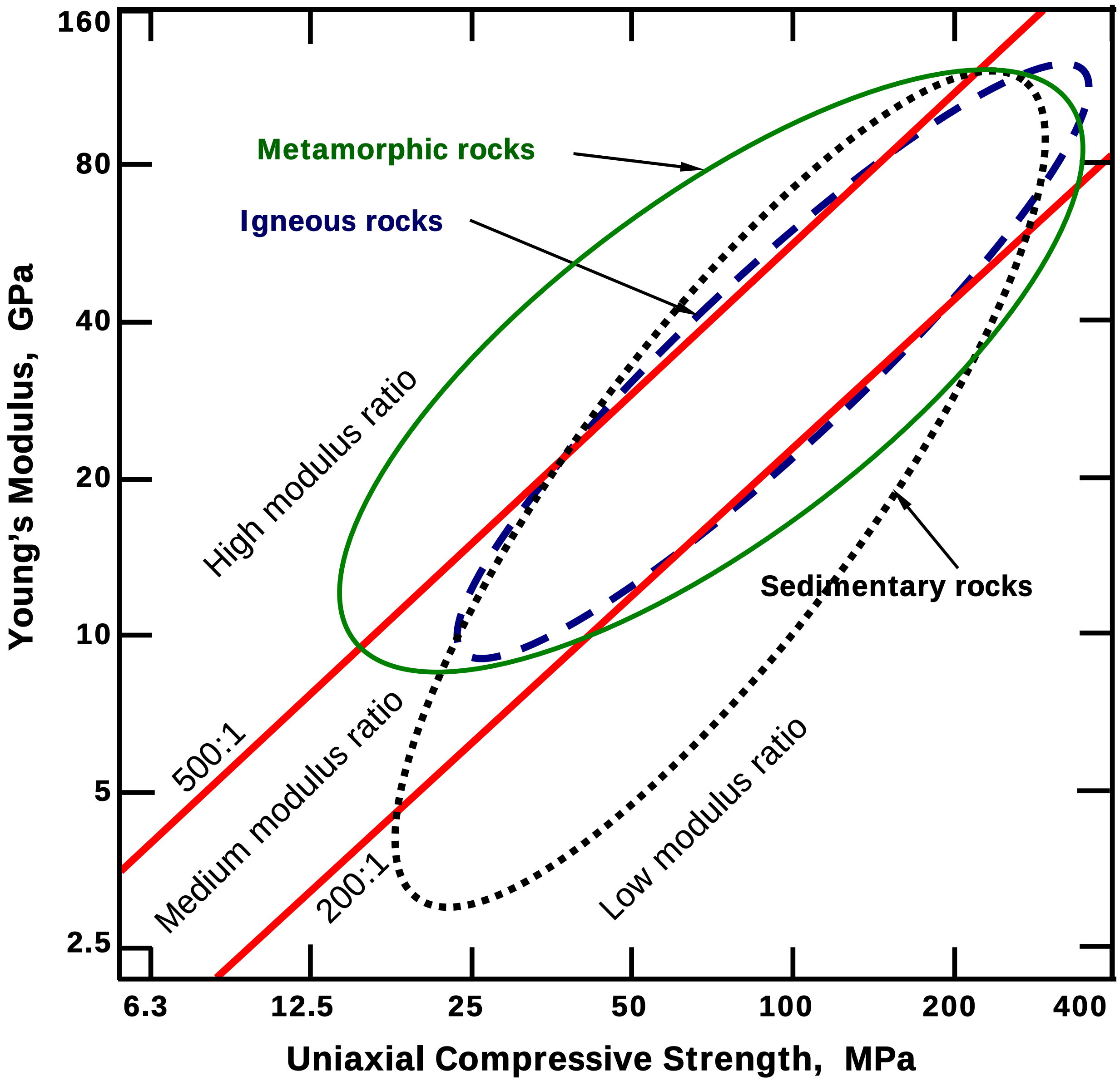
Strength and Deformation

Modulus of Elasticity and Poisson’s Ratio

Modulus of Elasticity and Poisson’s Ratio can be experimentally determined from the stress-strain curve. They seem to be unaffected by change of confining pressure.

High strength rocks also tend to have high Young's modulus, depending on rock type and other factors.

For most rocks, the Poisson’s ratio is between 0.15 and 0.4.



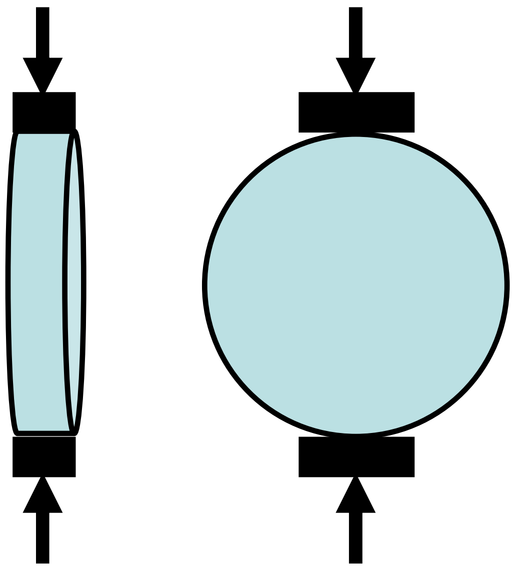
160
80
40
20
10
5
2.5
6.3
12.5
25
50
100
200
400
Young's Modulus, GPa
Uniaxial Compressive Strength, MPa
Metamorphic rocks
Igneous rocks
Sedimentary rocks
High modulus ratio
Medium modulus ratio
Low modulus ratio
500:1
200:1

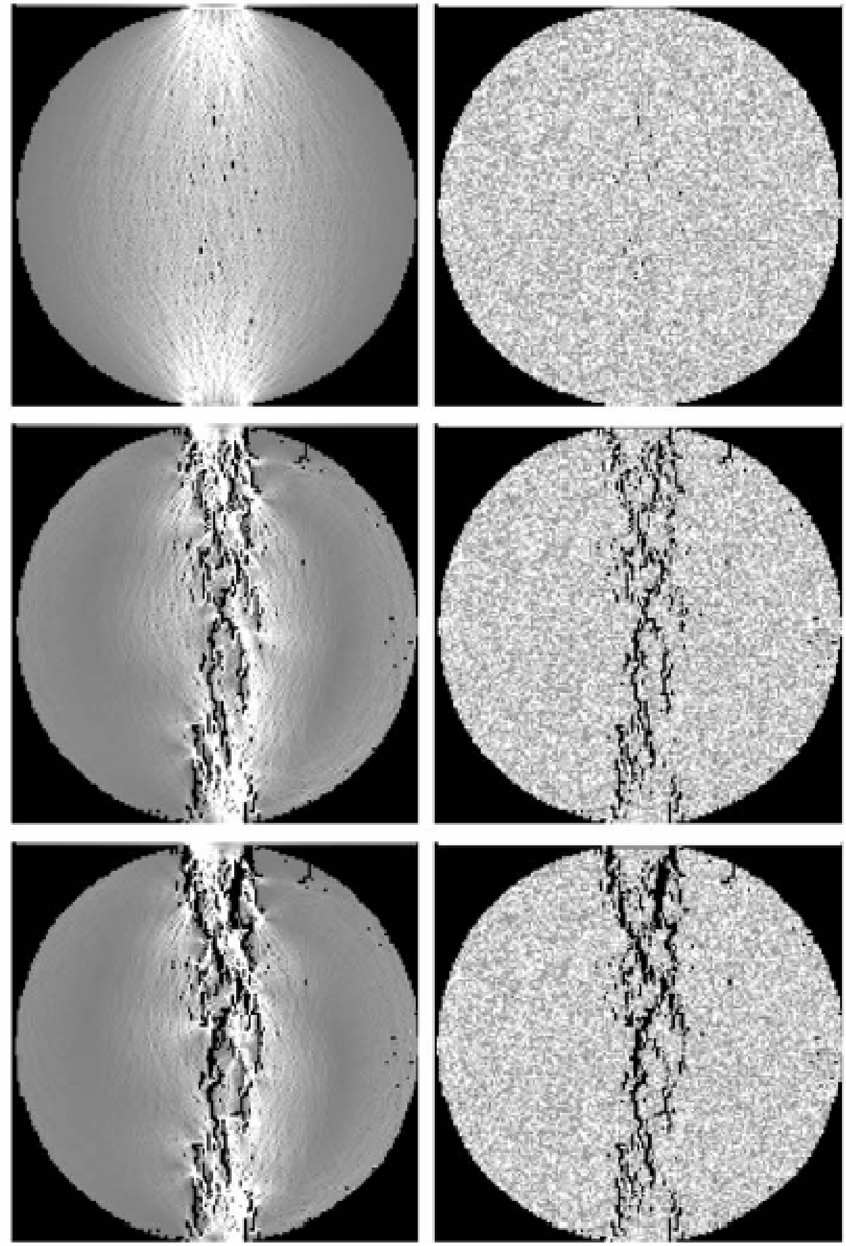
Strength and Deformation

Tensile Strength

Rock material generally has a low tensile strength, due to the pre-exiting microcracks in the rock material. The existence of microcracks may also be the cause of rock failing suddenly in tension with a small strain.

Rock material tensile strength can be obtained from several types of tests. The most common tensile test is the Brazilian tests.





Strength and Deformation

Shear Strength

Rock resists shear stress by two internal mechanisms, cohesion and internal friction. Cohesion is a measure of internal bonding of the rock material. Internal friction is caused by contact between particles, and is defined by the internal friction angle.

Shear strength of rock material can be determined by direct shear test and by triaxial compression tests.

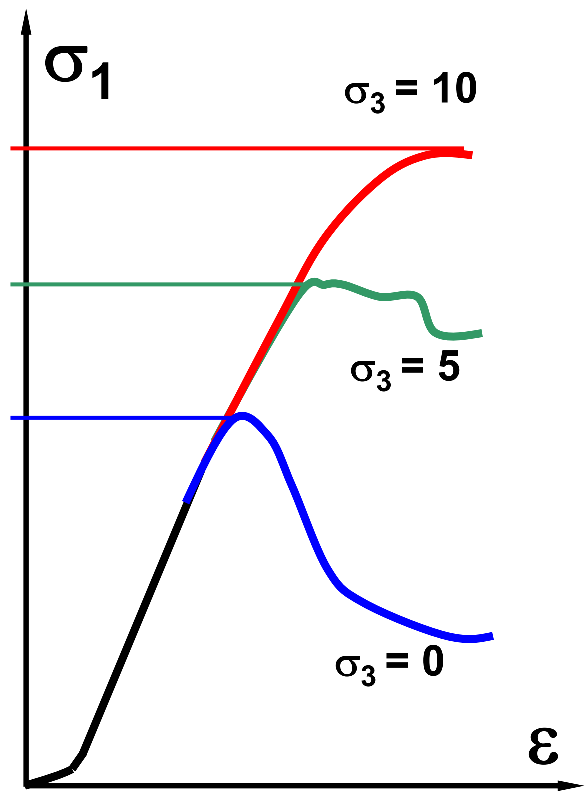
Strength and Deformation

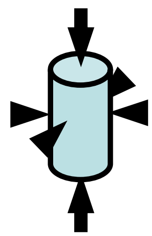
From a series of triaxial tests, peak stresses (σ_1) are obtained at various lateral stresses (σ_3). By plotting Mohr circles, the shear envelope is defined and gives the cohesion and internal friction angle.

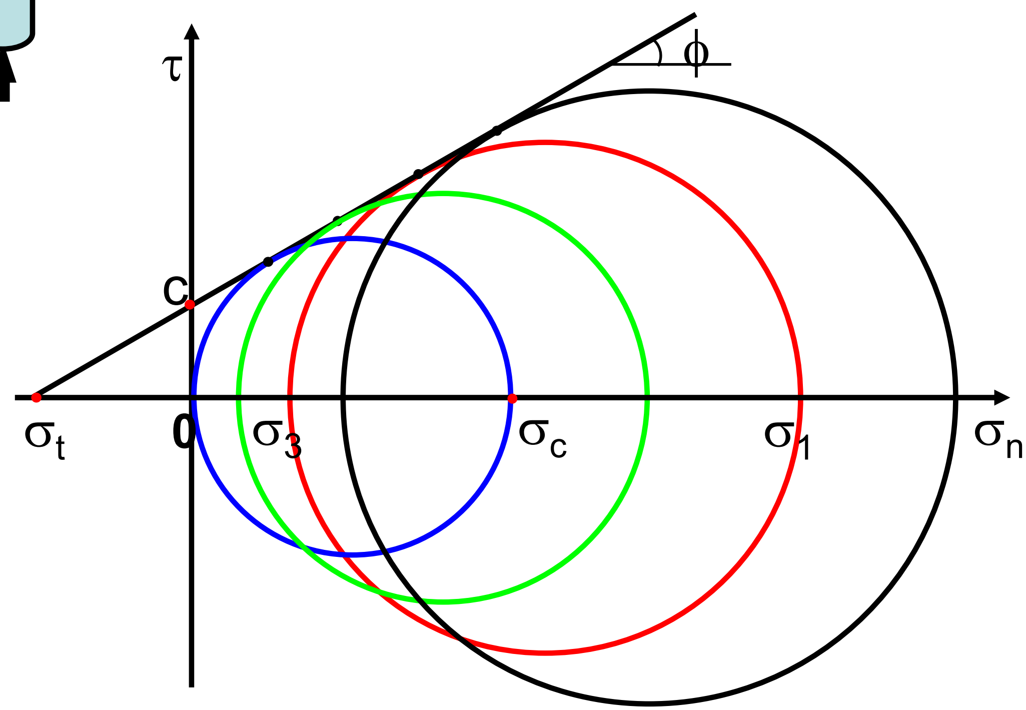
Test	σ_3 (MPa)	σ_1 (MPa)
1	0	41.2
2	1	52.6
3	3	74.1
4	5	90.3
5	10	122
6	15	151
7	20	172

Shear Strength from Triaxial Tests

Strength and Deformation







Strength and Deformation

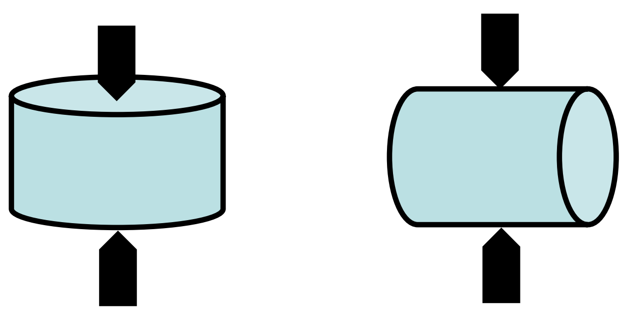
Rock	UC Strength (MPa)	Tensile Strength (MPa)
Granite	100 – 300	7 – 25
Dolerite	100 – 350	7 – 30
Gabbro	150 – 250	7 – 30
Basalt	100 – 350	10 – 30
Sandstone	20 – 170	4 – 25
Shale	5 – 100	2 – 10
Dolomite	20 – 120	6 – 15
Limestone	30 – 250	6 – 25
Gneiss	100 – 250	7 – 20
Slate	50 – 180	7 – 20
Marble	50 – 200	7 – 20
Quartzite	150 – 300	5 – 20

Compressive, Tensile and Shear Strengths

Tensile and shear strengths are important as rock fails mostly in tension and in shear, even the loading may appears to be compression. Rocks generally have high compressive strength so failure in pure compression is not common.

Point Load Index

Point load test is a simple index test for rock material. It gives the standard point load index, $I_{s(50)}$.



Granite	5 – 15
Gabbro	6 – 15
Andesite	10 – 15
Basalt	9 – 15
Sandstone	1 – 8
Mudstone	0.1 – 6
Limestone	3 – 7
Gneiss	5 – 15
Schist	5 – 10
Slate	1 – 9
Marble	4 – 12
Quartzite	5 – 15

Strength and Deformation

Correlation between Point Load Index and Strengths

$\sigma_c \approx 22\ I_{s(50)}$
Correction factor can vary between 10 and 30.

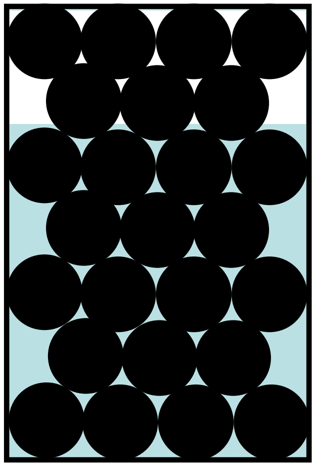
$\sigma_t \approx 1.25\ I_{s(50)}$

$I_{s(50)}$ should be used as an independent strength index.

Physical and Engineering Properties

Density, Porosity and Water Content

They are standard physical properties.



Density = Bulk mass / Bulk volume

Porosity = Non-solid volume / Bulk volume

Water content = Volume of water / Bulk volume

Physical and Engineering Properties

Dry density of rock material is generally between 2.5 and 2.8 g/cm^3. High density generally means low porosity.

Porosity is generally low for crystalline rocks, e.g., granite (<5%) and can be high for clastic sedimentary rocks, e.g., sandstone (up to 50%). Porosity affects permeability.

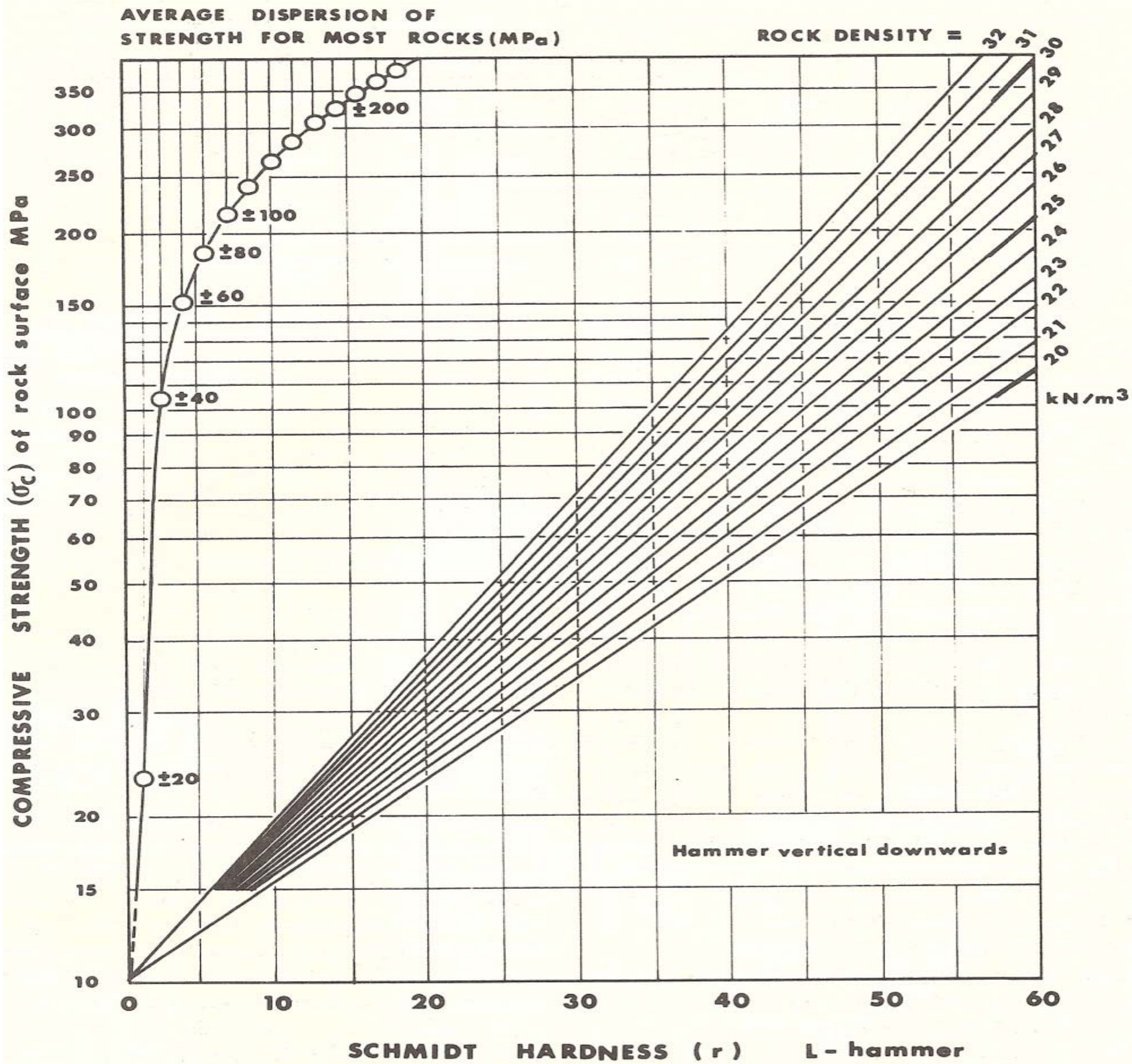
Water content depends on saturation. Wet rock tends to have slightly lower strength.

Physical and Engineering Properties

Hardness

Hardness is the characteristic of a solid material to resist permanent deformation. Rock material hardness depends on several factors, including mineral composition and density. A typical measure is the Schmidt rebound hardness number.

Schmidt hardness can be correlated to rock strength.



AVERAGE DISPERSION OF STRENGTH FOR MOST ROCKS (MPa)
ROCK DENSITY = 32 31 30 29 28 27 26 25 24 23 22 21 20 kN/m3
±200
±100
±80
±60
±40
±20
350
300
250
200
150
100
90
80
70
60
50
40
30
20
15
10
COMPRESSIVE STRENGTH (σ_c) of rock surface MPa
Hammer vertical downwards
0 10 20 30 40 50 60
SCHMIDT HARDNESS (r) L - hammer

Physical and Engineering Properties

Abrasivity

Abrasivity measures the abrasiveness of a rock materials against other materials, e.g., steel.

Abrasivity is highly influenced by the amount of quartz mineral in the rock material. The higher quartz content gives higher abrasivity.

Abrasivity are measured by tests, e.g., Cerchar test gives Cerchar Abrasivitiy Index (CAI).

Physical and Engineering Properties

Granite	4.5 – 5.3
Diorite	4.2 – 5.0
Andesite	2.7 – 3.8
Basalt	2.0 – 3.5
Sandstone	1.5 – 3.5, 2.8 – 4.2
Shale	0.6 – 1.8
Limestone	1.0 – 2.5
Gneiss	3.5 – 5.3
Slate	2.3 – 4.2
Quartzite	4.3 – 5.9

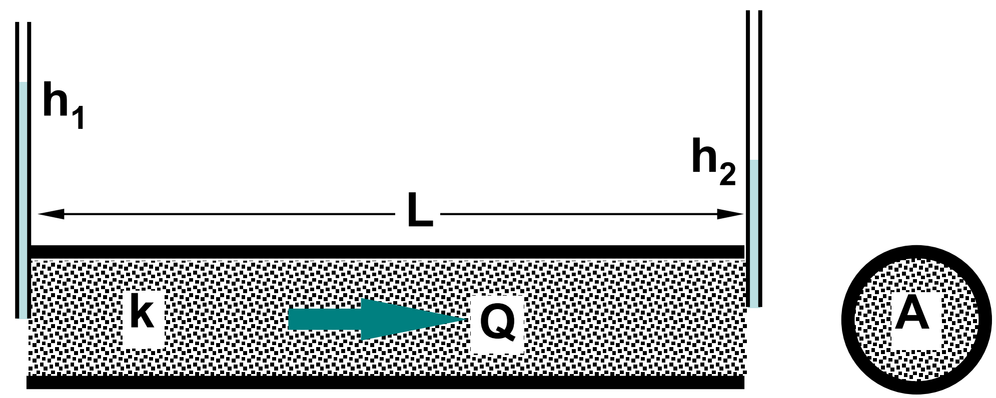
Cerchar Abrasivitiy Index (CAI)

Physical and Engineering Properties

Permeability

Permeability is a measure of the ability of a material to transmit fluids. It is given by the Darcy's law,

$$Q = A\,k\,(h_1 - h_2)/L$$



Q = Flow rate
k = Coefficient of permeability
A = cross section area
L = length
h_1, h_2 = hydraulic head

Physical and Engineering Properties

Permeability

Most rocks have very low permeability. Permeability of rock material is governed by porosity. Porous rocks such as sandstones usually have high permeability while granites have low permeability. Permeability of rock materials, except for those porous one, has limited interests. In the rock mass, flow is concentrated in fractures.

Physical and Engineering Properties

Wave Velocity

Two types of wave are often used in velocity measurements: longitudinal (P) wave and shear (S) wave. P wave is the fastest travelled wave and therefore is the most commonly used one in wave velocity measurements.

Wave velocity is related to the compaction degree (density and porosity) of the rock material. A well compacted rock has generally high velocity as the grains are in good contact and wave travels through solid grains.

P-wave velocity of igneous rocks, gneiss and quartzite is 5000-7000 m/s, and of shale, sandstone and conglomerate 3000-5000 m/s.

Physical and Engineering Properties

Wave Velocities and Deformation Modulus

Wave velocity can be used to estimate the modulus of the rock material. The modulus estimated is generally slightly higher than the modulus determined from static tests.

Elastic modulus $E_s = \rho v_s^2$ (GPa), (g/cm^3), (km/s)

Shear modulus $G_s = \rho v_s^2$ (GPa), (g/cm^3), (km/s)

Poisson's ration $\nu_s = [1-2(v_s/v_p)^2] / \{2[1-(v_s/v_p)^2]\}$